DSF-05/2005

LEPTON MIXING AND SEESAW MECHANISM

D. Falcone

Dipartimento di Scienze Fisiche, Università di Napoli, Via Cintia, Napoli, Italy

In the context of a typical model for fermion mass matrices, possibly based on the horizontal U(2) symmetry, we explore the effect of the type II seesaw mechanism on lepton mixings. We find that the combined contribution of type I and type II terms is able to explain the large but not maximal 1-2 mixing and the near maximal 2-3 mixing, while the 1-3 mixing angle is predicted to be small.

I. INTRODUCTION

By now there is convincing evidence for neutrino oscillation, which imply neutrino mass and lepton mixing. Thus, the framework seems analogous to that of quark mass and mixing. However, there are two major differences. First, neutrino mass is very small, with respect to quark (and charged lepton) mass. Second, lepton mixing can be large and even maximal, while quark mixing is small. Both features can be explained by means of the (type I) seesaw mechanism [1].

In fact, for a single fermion generation, the effective neutrino mass m_ν is given by $m_\nu \simeq (m_D/m_R)m_D$, where the Dirac mass m_D is of the order of the quark (or charged lepton) mass, and the right-handed Majorana mass m_R is of the order of the unification or intermediate mass scale. As a result, m_ν is very small with respect to m_D.

For two and three fermion generations, the effective neutrino mass matrix M_ν is given by the formula

$$M_\nu \simeq M_D^T M_R^{-1} M_D, \tag{1}$$

so that large neutrino mixing can be generated from a nearly diagonal M_D: A strong mass hierarchy or large offdiagonal elements in M_R are required [2, 3]. A small contribution to the lepton mixing from the charged lepton mass matrix M_e is also expected, which could be important to understand the deviation from maximal mixing [4].

The type I seesaw mechanism is based on the introduction, within the minimal standard model, of three heavy right-handed neutrinos. However, small neutrino masses can be generated also by the inclusion of a heavy Higgs triplet [5]. In this case the neutrino mass matrix is given by $M_\nu = M_L = Y_L v_L$, where Y_L is a Yukawa matrix and v_L is the v.e.v. of the triplet, which can be written as $v_L = \gamma v^2/m_T$, with v the v.e.v. of a standard Higgs doublet, m_T the triplet mass, and γ a coefficient related to the coupling between the doublet and the triplet. Then, v_L is small with respect to v, but large mixing in M_ν is achieved by hand. Instead, from the type I seesaw formula (1) we get $M_\nu \simeq Y_D^T M_R^{-1} Y_D v^2$, so that large mixing can be generated from the structure of both matrices Y_D and M_R.

More generally, we can write also a type II seesaw formula by adding to the usual type I term (1) the triplet (or type II) term, so that

$$M_\nu \simeq M_D^T M_R^{-1} M_D + M_L. \tag{2}$$

This is called type II seesaw mechanism. Large mixing in M_L should be introduced by hand. Nevertheless, the structure and the scale of M_L could produce an effect on neutrino mixing and even explain the deviation from maximal mixing.

In the present paper we take as starting point a horizontal $U(2)$ inspired model for fermion mass matrices [6], in order to explore the type II seesaw mechanism. We aim to

study in particular the deviation from maximal mixing, the contribution of M_e and M_L, and the value of the small mixing angle θ_{13}. Contrary to Ref.[7], we consider the type I term, and not M_L, as the basic neutrino mass term.

II. LEPTON MIXING

The lepton mixing matrix U is defined by $U = U_e^\dagger U_\nu$, where U_e and U_ν diagonalize M_e and M_ν, respectively. Since U_{23} is near maximal, U_{12} is large but not maximal, and U_{13} is small, then U has the approximate form

$$U \simeq \begin{pmatrix} c & s & \epsilon \\ -\frac{1}{\sqrt{2}}(s+c\epsilon) & \frac{1}{\sqrt{2}}(c-s\epsilon) & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}}(s-c\epsilon) & -\frac{1}{\sqrt{2}}(c+s\epsilon) & \frac{1}{\sqrt{2}} \end{pmatrix}, \tag{3}$$

with $s \simeq \frac{1}{\sqrt{3}}$ and $\epsilon < 0.2$. In fact, assuming a standard parametrization, we have

$$0.48 < \sin\theta_{12} < 0.62, \tag{4}$$

$$0.56 < \sin\theta_{23} < 0.84, \tag{5}$$

with central values $\sin\theta_{12} = 0.55$, $\sin\theta_{23} = 0.70$, respectively, and $\sin\theta_{13} < 0.23$. For a recent clear account of neutrino phenomenology, see Ref.[8].

III. MASS MATRICES

Horizontal symmetries, which relate particles of different generations, have been often used to explain the hierarchical pattern of mass matrices [9]. For instance, the Abelian $U(1)$ symmetry and the non-Abelian $U(2)$ symmetry. The last one is based on the assumption that the three generations transform as a doublet plus a singlet: $\psi_a + \psi_3$. Then the $U(2)$ symmetry is broken down to the $U(1)$ symmetry and again to nothing, generating specific forms of mass matrices.

According to the $U(2)$ model described in Ref.[6], we have the following approximate expression for neutrino mass matrices,

$$M_D \sim \begin{pmatrix} \lambda^{12} & \lambda^6 & \lambda^{10} \\ -\lambda^6 & \lambda^4 & \lambda^4 \\ \lambda^{10} & \lambda^4 & 1 \end{pmatrix} m_t, \tag{6}$$

$$M_R \sim \begin{pmatrix} \lambda^{12} & \lambda^{10} & \lambda^6 \\ \lambda^{10} & \lambda^8 & \lambda^4 \\ \lambda^6 & \lambda^4 & 1 \end{pmatrix} m_R, \tag{7}$$

where $\lambda = 0.2$ is the Cabibbo parameter, and $m_t \simeq v$. Moreover, we have

$$M_e \sim \begin{pmatrix} \lambda^6 & \lambda^3 & \lambda^5 \\ -\lambda^3 & \lambda^2 & \lambda^2 \\ \lambda^5 & \lambda^2 & 1 \end{pmatrix} m_b. \tag{8}$$

Matrices (6) and (8) were obtained, on the phenomenological side, by inserting the quark mass hierarchy into a widely adopted form of quark mass matrices [10], and, on the theoretical side, by means of the broken $U(2)$ family symmetry [11]. Simple quark-lepton relations, $M_e \sim M_d$ and $M_D \sim M_u$, were also assumed. Matrix (7) was obtained by inverting the (type I) seesaw formula, and was found consistent with the broken $U(2)$ horizontal symmetry.

Then $M_\nu^I \simeq M_D^T M_R^{-1} M_D$ may be given by the matrix

$$M_\nu^I \sim \begin{pmatrix} \lambda^4 & \lambda^2 & -\lambda^2 \\ \lambda^2 & 1 & 1 \\ -\lambda^2 & 1 & 1 \end{pmatrix} \frac{m_t^2}{m_R}, \tag{9}$$

which corresponds to a normal mass hierarchy of neutrinos. We will take also

$$M_\nu^{II} = M_L = \frac{m_L}{m_R} M_R. \tag{10}$$

The last relation can be motivated by assuming that the structure of both matrices M_R and M_L is generated by coupling with the same flavon fields. It is just a conjecture: since the Dirac mass matrices have similar structures, we may think that both the Majorana mass matrices have one specific structure.

IV. MODEL EXPLORATION

Since the 2-3 sector of M_ν^I could not be exactly democratic, we consider the following form, as a perturbation of (9),

$$M_\nu^I \simeq \begin{pmatrix} \lambda^4 & \lambda^2 & -\lambda^2 \\ \lambda^2 & 1+\frac{\lambda^n}{2} & 1-\frac{\lambda^n}{2} \\ -\lambda^2 & 1-\frac{\lambda^n}{2} & 1+\frac{\lambda^n}{2} \end{pmatrix} \frac{m_t^2}{m_R}, \tag{11}$$

which has 2-3 symmetry [12] and hence near maximal 2-3 mixing, with $n = 1, 2, 3, 4$. Then, for $n = 4$ we get the bimaximal mixing, that is $s = 1/\sqrt{2}$ and $\epsilon = 0$. For $n = 3$ we have $\tan 2\theta_{12} \simeq 2\sqrt{2}/\lambda$ or $\sin\theta_{12} \simeq 0.68$. For $n = 2$, $\tan 2\theta_{12} \simeq 2\sqrt{2}$ or $\sin\theta_{12} \simeq 0.58$. For $n = 1$, $\tan 2\theta_{12} \simeq 2\sqrt{2}\lambda$ or $\sin\theta_{12} \simeq 0.25$.

The contribution from M_ν^{II}, with respect to M_ν^I, will be parametrized by the ratio

$$k = \frac{m_L m_R}{v^2} = \gamma\,\frac{m_R}{m_T} \tag{12}$$

and leads to a decrease of the mixings. We perform a numerical analysis. For useful formulas see the appendix of Ref.[13].

Now we consider the contribution to lepton mixing from M_e [14, 15]. This is similar to the Wolfenstein parametrization of the CKM matrix [16]. We easily obtain

$$\sin\theta_{12} \simeq \sin\theta_{12}^\nu - \frac{\lambda}{2}, \tag{13}$$

$$\sin\theta_{23} \simeq \frac{1}{\sqrt{2}}\left(1 - \lambda^2\right), \tag{14}$$

$$\sin\theta_{13} \simeq -\frac{\lambda}{\sqrt{2}}. \tag{15}$$

If we include in the element 2-2 of M_e the -3 factor by Georgi and Jarlskog (GJ) [17], which reproduces better m_e and m_μ, we get instead

$$\sin\theta_{12} \simeq \sin\theta_{12}^\nu + \frac{\lambda}{6}, \tag{16}$$

$$\sin\theta_{23} \simeq \frac{1}{\sqrt{2}}\left(1 - \lambda^2\right),$$

$$\sin\theta_{13} \simeq \frac{\lambda}{3\sqrt{2}}. \tag{17}$$

Taking into account the three contributions, M_ν^I, M_ν^{II}, and M_e, in this order, and matching with the allowed ranges of lepton mixings, reported in section II, we get the following results.

Case $n = 4$ requires $0 \le k < 0.05$, or $0.08 < k < 0.18$ for the GJ option.

Case $n = 3$ requires $0 \le k < 0.04$, or $0.06 < k < 0.16$ for the GJ option.

Case $n = 2$ is reliable only for the GJ choice with $0 \le k < 0.10$.

Case $n = 1$ is not reliable at all.

V. DISCUSSION

We see that the contribution from M_ν^{II} is necessary for the cases $n = 4$ and $n = 3$ with the GJ option. However, its impact is most important only on the 1-2 mixing, while it is of the order 10^{-2} on the 2-3 mixing and 10^{-3} on the 1-3 mixing. Therefore, in our framework, near maximal 2-3 mixing can be ascribed mainly to M_ν^I, and the small but not zero 1-3 mixing mainly to M_e. Instead, three mass matrices contribute to the 1-2 mixing, thus providing naturally a large but not maximal value.

It has been pointed out [15] that the inclusion of the -3 factor by GJ is not consistent with the observed quark-lepton complementarity $\theta_{12} + \theta_C \simeq \pi/4$. Based on the previous section, we argue that the contribution of the triplet seesaw is able to correct such a disagreement.

We have studied a type I term of the form (11), and a type II term of the form (10),(7). For other choices considered in the literature, see for instance Ref.[18].

[1] P. Minkowski, Phys. Lett. B 67 (1977) 421
M. Gell-Mann, P. Ramond and R. Slansky, in *Supergravity*, eds. P. van Nieuwenhuizen and D. Freedman (North Holland, Amsterdam, 1979)
T.Yanagida, in *Proceedings of the Workshop on Unified Theories and Baryon Number in the Universe*, eds. O. Sawada and A. Sugamoto (KEK, Tsukuba, 1979)
S.L. Glashow, in *Quarks and Leptons*, eds. M. Levy et al. (Plenum, New York, 1980)
R.N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. 44 (1980) 912
[2] A.Yu. Smirnov, Phys. Rev. D 48 (1993) 3264
[3] M. Jezabek and Y. Sumino, Phys. Lett. B 440 (1998) 327
G. Altarelli, F. Feruglio and I. Masina, Phys. Lett. B 472 (2000) 382
D. Falcone, Phys. Lett. B 479 (2000) 1
[4] M. Jezabek and Y. Sumino, Phys. Lett. B 457 (1999) 139
[5] G. Lazarides, Q. Shafi and C. Wetterich, Nucl. Phys. B 181 (1981) 287
R.N. Mohapatra and G. Senjanovic, Phys. Rev. D 23 (1981) 165
[6] D. Falcone, Phys. Rev. D 64 (2001) 117302
[7] W. Rodejohann, Phys. Rev. D 70 (2004) 073010
[8] G. Altarelli and F. Feruglio, New J. Phys. 6 (2004) 106
[9] C. Froggatt and H. Nielsen, Nucl. Phys. B 147 (1979) 277
M. Leurer, Y. Nir and N. Seiberg, Nucl. Phys. B 398 (1993) 319
[10] H. Nishiura, K. Matsuda and T. Fukuyama, Phys. Rev. D 60 (1999) 013006
H. Fritzsch and Z. Xing, Nucl. Phys. B 556 (1999) 49
[11] R. Barbieri, L. Giusti, L. Hall and A. Romanino, Nucl. Phys. B 550 (1999) 32
R. Barbieri, P. Creminelli and A. Romanino, Nucl. Phys. B 559 (1999) 17
C.D. Carone and L.J. Hall, Phys. Rev. D 56 (1997) 4198
[12] C.S. Lam, Phys. Lett. B 507 (2001) 214
[13] W. Rodejohann, Eur. Phys. J. C 32 (2004) 235
[14] C. Giunti and M. Tanimoto, Phys. Rev. D 66 (2002) 053013
[15] J. Ferrandis and S. Pakvasa, Phys. Lett. B 603 (2004) 184; Phys. Rev. D 71 (2005) 033004
[16] L. Wolfenstein, Phys. Rev. Lett. 51 (1983) 1945
[17] H. Georgi and C. Jarlskog, Phys. Lett. B 86 (1979) 297
[18] S. Antusch and S. King, Nucl. Phys. B 705 (2005) 239
W. Rodejohann and Z. Xing, Phys. Lett. B 601 (2004) 176
W. Guo, Phys. Rev. D 70 (2004) 053009